Assignment 3

(Deadline: Dec. 26. **Late submission will be considered as 0 point.**)

1. **(30 points)**

(1) For a given univariate Gaussian distribution, show that $\mathbb{E}[x] = \mu, \mathrm{var}[x] = \sigma^2$.

(2) For a given univariate Gaussian distribution, show that the maximum of the Gaussian distribution is obtained when $x = \mu$.

2. **(30 points)**

Suppose that we have three coloured boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange, and 0 limes, and box g contains 3 apples, 3 oranges, and 4 limes. If a box is chosen at random with probabilities $p(r) = 0.2, p(b) = 0.2, p(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple? If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?

3. **(40 points)**

Define an HMM model with three states $\{A, B, C\}$ and observations $\{0,1,2\}$. The initial stable probabilities are $\pi_A = 1$, and $\pi_B = \pi_C = 0$. The transition and emission probabilities are as follows:

	A	B	C	0	1	2
A	0.2	0.8	0.0	0.8	0.2	0.0
B	0.0	0.8	0.2	0.0	0.6	0.4
C	0.4	0.0	0.6	0.2	0.0	0.8

(1) Draw the state diagram of this HMM and show the transition probabilities.

(2) Give all state paths with non-zero probability for the sequence $O = 0,1,2$.

(3) What is $P(O)$?