EE211: Robotic Perception and Intelligence

Lecture 7 Inertial Sensing, GPS and Odometry

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Outline

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Concept of Odometry

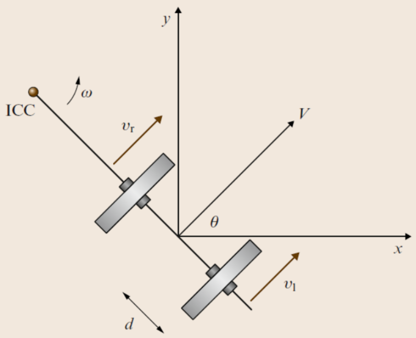
- Contraction of the Greek words hodos meaning travel or journey, and metron meaning measure.
- Perhaps the earliest reference from Ten Books on Architecture of Vitruvius.
- A useful invention of the greatest ingenuity, transmitted by our predecessors, which enables us, while sitting in a carriage on the road or sailing by sea, to know how many miles of a journey we have accomplished.

Springer Handbook of Robotics.

Odometry in Autonomous Vehicles

- The use of data from the actuators (wheels, treads, etc.) to estimate the overall motion of the vehicle.
- Develop a mathematical model of how commanded motions of the vehicles wheels/joints/etc. induce motion of the vehicle itself, and then to integrate these commanded motions over time in order to develop a model of the pose of the vehicle as a function of time.
- Dead Reckoning or Deductive Reckoning: use odometry information to estimate the pose of the vehicle as a function of time.

Example: Differential Drive Vehicle



- Two driveable wheels which are independently controllable and which are mounted along a common axis.
- ICC – instantaneous center of curvature.

Example: Differential Drive Vehicle

- If the ground contact speeds of the left and right wheels are v_l and v_r respectively, and the wheels are separated by a distance $2d$, then

$$\omega(R + d) = v_r, \ \omega(R - d) = v_l.$$

- Rearrange these two equations to solve for ω the rate of rotation about the ICC and R the distance from the center of the robot to the ICC

$$\omega = (v_r - v_l)/2d,$$
$$R = d(v_r + v_l)/(v_r - v_l).$$

- The instantaneous velocity of the point midway between the robot's wheels is given by

$$V = \omega R.$$

Example: Differential Drive Vehicle

- Using the point midway between the wheels as the origin of the robot, and writing θ as the orientation of the robot with respect to the x-axis of a global Cartesian coordinate system obtains

$$x(t) = \int V(t)\cos[\theta(t)]\mathrm{d}t,$$
$$y(t) = \int V(t)\sin[\theta(t)]\mathrm{d}t,$$
$$\theta(t) = \int \omega(t)\mathrm{d}t.$$

- Given control inputs (v_l and v_r) and some initial state estimate, we can estimate where an idealized robot using this motion model will be at any time t.

Further Discussion

- Errors in the modelling (incorrect estimations of wheel/vehicle size).
- Uncertainty about the control inputs.
- Realities of the motor controller (errors between commanded wheel rotation and true rotation).
- Errors in the physical modelling of the robot (wheel compaction, ground compaction, wheel slippage, non-zero tire width).
- All above introduce an error between the dead reckoning estimate of the vehicle motion and its true motion.
- Pose maintenance for the vehicle: require the integration of the dead reckoning estimate with estimates obtained from other sensor systems.

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Gyroscopic Systems

- Measure changes in vehicle orientation by taking advantage of physical laws that produce predictable effects under rotation.
- Rely on the principle of the conservation of angular momentum.
- Angular momentum is the tendency of a rotating object to keep rotating at the same angular speed about the same axis of rotation in the absence of an external torque.
- The angular momentum L of an object with moment of inertia I rotating at angular speed ω is given by

$$L = I \times \omega.$$

Working Principle of Gyroscopic Systems

https://haokan.baidu.com/v?vid=4917207119914294731

Outline

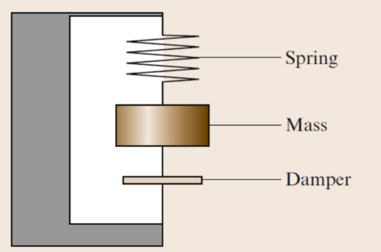
Inertial Sensors

- Gyroscopes: measure changes in orientation of a robot.
- Accelerometers: measure external forces acting on the robot.
- Use one of a number of different mechanisms to transduce external forces into a computer readable signal.

Mechanical Accelerometer

- A spring-mass-damper system with some mechanism for external monitoring.
- Accelerometers: measure external forces acting on the robot.
- Assuming an ideal spring with a force proportional to its displacement, the external forces balance the internal ones

$$\begin{aligned} F_{applied} &= F_{intertial} + F_{damping} + F_{spring} \\ &= m\ddot{x} + c\dot{x} + kx \end{aligned}$$



Mechanical Accelerometer

- This equation can be solved to show that depending on the size of the damping coefficient relative to the expected external force and the mass, the system can be made to reach a stable final value in a reasonably short period of time whenever a static force is presented.

$$\begin{aligned} F_{applied} &= F_{intertial} + F_{damping} + F_{spring} \\ &= m\ddot{x} + c\dot{x} + kx \end{aligned}$$

  where c is the damping coefficient.
- Disadvantages: (1) Slow convergence on a final measurement; (2) Non-ideal performance of the spring; (3) Particularly sensitive to vibration.

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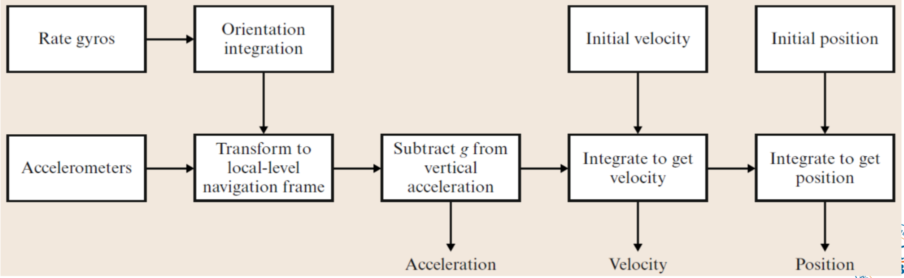
Inertial Measurement Unit

- IMU: a device that utilizes measurement systems such as gyroscopes and accelerometers to estimate the relative position, velocity and acceleration of a vehicle in motion.
- Historically an IMU is self-contained and provides this estimate without reference to external references, however the definition has become less precise in recent years and now it is common to have the term IMU also refer to systems which do include such external references.

IMU Block Diagram

- 3 orthogonal gyroscopes $\rightarrow$ vehicle orientation θ.
- 3 accelerometers $\rightarrow$ instantaneous vehicle acceleration a.
- This data is then transformed via the current estimate of the vehicle orientation relative to gravity, so that the gravity vector can be estimated and extracted from the measurement.
- The resulting acceleration $\rightarrow$ vehicle velocity v $\rightarrow$ position r .



IMU Error Corrections

- IMUs are extremely sensitive to measurement errors in the underlying gyroscopes and accelerometers.
- Given a sufficiently long period of operation all IMUs eventually drift and reference to some external measurement is required to correct this.
- For many field robots GPS has become an effective source for these external corrections.

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Overview

- GNSS: global navigation satellite system.
- GPS: global positioning system, most commonplace instance of GNSS.
- GPS provides 3D position estimate in the horizontal plane within about 20 meters.
- GPS is rapidly being superseded (via augmentation) by systems that add related technologies to basic GPS reception.

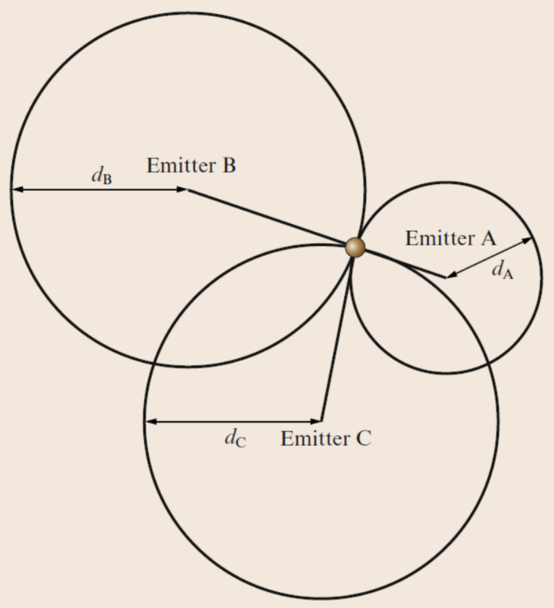
Overview

- GPS: NAVSTAR satellite system deployed and maintained by Air Force Space Command, United States.
- GLONASS: globalnaya navigatsionnaya sputnikovaya sistema, operated by the Russian government.
- Galileo: deployed by the European Union.
- Beidou: developed by China.
- Japanese quasi-zenith satellite system (QZSS), the Indian regional navigational satellite system (IRNSS) system.
- Many modern GPS receivers and cell phones can access different systems.
- Starlink: a satellite internet constellation operated by SpaceX.

Working Principle of GPS

- Based on received radio signals transmitted by an ensemble of satellites orbiting the earth.
- A base constellation of 24 orbiting satellites along with up to 6 supplementary additional satellites that are also operational.
- The orbits are selected so that from almost any point of the earth's surface there will always be four or more satellites directly visible – a criterion for obtaining a GPS position estimate.

Working Principle of GPS



Performance Factors

- Satellite transmission accuracy.
- Environmental conditions.
- Interactions with ground-based obstacles.
- Receiver properties.

Enhanced GPS

- WAAS: Wide Area Augmentation System, based around a supplementary signal that can be received by GPS receivers to improve their accuracy. from 10-12m with GPS alone to between 1 and 2m.
- The WAAS signal contains corrections for the GPS signal that reduce the effects of errors due to timing errors, satellite position corrections, and local perturbations due to variations in the ionosphere.
- Correction terms are estimated by ground-based stations at fixed and accurately-known positions and uplinked to satellites which broadcast them to suitably enabled GPS receivers.

Enhanced GPS

- DGPS: Differential GPS, using a nearby GPS receiver located at a known accurately-surveyed position.
- Use the same principles as WAAS but on a local scale without resorting to the use of satellite uplinks.
- Since the error varies as a function of position on the earth, the effectiveness of the correction degrades with distance, typically with a maximum effective range of a couple of hundred miles.
- Commercial DGPS solutions also exist akin to WAAS.

Enhanced GPS

- RTK: Real-Time Kinematic Positioning, can achieve horizontal positioning accuracy on the order of 1cm.
- Augments standard differential GPS technology by enhancing the effective resolution of the pseudo-range distance estimates using corrections obtained by a base station.
- RTK estimates the number of carrier cycles in the communication path between a GPS satellite and each of the base station and the receiver, and thus is essentially dependent on the phase of the signal.

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Drawbacks of GPS

- Cannot directly obtain information about vehicle orientation.
- Unable to provide continuous independent estimates of position. Estimates are only available at distinct time instances with (for inexpensive receivers at least) considerable delays between measurements. A continuous estimate of pose requires estimation of pose between GPS readings.
- Not always possible to obtain a GPS fix. Local geography (e.g., mountains, buildings, trees) or an overhead cover that is opaque to radio signals (e.g., indoors, underwater) can block the signal entirely.

GPS-IMU Integration

- Expressed as an extended Kalman filter estimation process.
- Essentially the IMU data is used to bridge between solid GPS measurements and is combined in a least squares optimal sense with the GPS data when both are available.
- Further reading: J. Rios, E. White: Low cost solid state GPS/INS package, Proc. Inst. Navig. Conf. (2000)

Q & A

1. Odometry
2. Gyroscopic Systems
3. Accelerometers
4. IMU Packages
5. Satellite-Based Positioning
6. GPS-IMU Integration